Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
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Centre Number

Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WMA14/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P4

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶







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**1.** The curve $C$ has equation

$$2x - 4y^2 + 3x^2y = 4x^2 + 8$$

The point $P(3, 2)$ lies on $C$.

Find the equation of the normal to $C$ at the point $P$, writing your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers to be found.

**(7)**

★ Implicit differentiation:

When differentiating $y$, multiply by $\frac{dy}{dx}$

$$2x - 4y^2 + 3x^2y = 4x^2 + 8$$

Product Rule:

$$\frac{d}{dx}(uv) = uv' + u'v$$

$$2 - 8y\frac{dy}{dx} + 6xy + 3x^2\frac{dy}{dx} = 8x \quad \text{M1M1A1}$$

$$3x^2\frac{dy}{dx} - 8y\frac{dy}{dx} = 8x - 6xy - 2$$ collect all $\frac{dy}{dx}$ terms on one side

$$\frac{dy}{dx}(3x^2 - 8y) = 8x - 6xy - 2$$ factor out $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{8x - 6xy - 2}{3x^2 - 8y}$$ Isolate $\frac{dy}{dx}$ on LHS

Use our found $\frac{dy}{dx}$ to get the gradient of the tangent at P:

$$\left.\frac{dy}{dx}\right|_{\substack{y=2\\x=3}} = \frac{8(3) - 6(3)(2) - 2}{3(3)^2 - 8(2)} = -\frac{14}{11}$$ m(tangent)

M1A1

$$m(\text{normal}) = -\frac{1}{m(\text{tangent})} \qquad \therefore\ m(\text{normal}) = \frac{11}{14}.$$

Use formula $y - y_1 = m(x - x_1)$ to get the equation of the line:

$m = \frac{11}{14}$, $P(3, 2)$:

$$y - 2 = \frac{11}{14}(x - 3) \quad \text{dM1}$$

$$14y - 28 = 11x - 33$$

$$0 = 11x - 14y - 5 \quad \text{A1}$$

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**Question 1 continued**

Q1

**(Total 7 marks)**







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**2.** Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x+5}} \qquad x > -\frac{5}{4}$$

for which $y = \frac{1}{3}$ at $x = -\frac{1}{4}$ giving your answer in the form $y = f(x)$

**(6)**

$$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x+5}}$$

$$\int \frac{1}{4y^2}\,dy = \int \frac{1}{\sqrt{4x+5}}\,dx$$ Separate Variables by grouping the like terms on one side B1

$$\frac{1}{4}\int y^{-2}\,dy = \int (4x+5)^{-\frac{1}{2}}\,dx$$ Integrate this

$$\frac{1}{4} \times -y^{-1} = (4x+5)^{\frac{1}{2}} \times \frac{1}{\frac{1}{2}} \times \frac{1}{4}$$ Reverse chain Rule: divide by derivative of the bracket

$$-\frac{1}{4y} = \frac{1}{2}(4x+5)^{\frac{1}{2}} + c$$ M1A1

Substitute the given $y=\frac{1}{3}$ and $x=-\frac{1}{4}$ to get the value of $+c$:

$$-\frac{1}{4\left(\frac{1}{3}\right)} = \frac{1}{2}\left(4 \times -\frac{1}{4}+5\right)^{\frac{1}{2}} + c$$

$$-\frac{3}{4} = \frac{1}{2}\sqrt{\cancel{4}}^{\,2} + c$$

$$-\frac{3}{4} - 1 = c \Rightarrow c = -\frac{7}{4}$$ dM1

Substitute c back into the integration result to get the particular solution:

$$-\frac{1}{4y} = \frac{1}{2}(4x+5)^{\frac{1}{2}} - \frac{7}{4}$$

$$\frac{1}{y} = -4\left(\frac{1}{2}(4x+5)^{\frac{1}{2}} - \frac{7}{4}\right)$$ multiply both sides by -4

$$\frac{1}{y} = -2(4x+5)^{\frac{1}{2}} + 7$$ expand

$$\left(\frac{1}{y}\right)^{-1} = \left(-2(4x+5)^{\frac{1}{2}} + 7\right)^{-1}$$ take the reciprocal on both sides to make y the subject

$$y = \frac{1}{7 - 2\sqrt{4x+5}}$$ found ddM1A1



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**Question 2 continued**

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**Question 2 continued**

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**Question 2 continued**

Q2

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**3.** $$\mathrm{g}(x) = \frac{3x^3 + 8x^2 - 3x - 6}{x(x+3)} \equiv Ax + B + \frac{C}{x} + \frac{D}{x+3}$$

(a) Find the values of the constants $A$, $B$, $C$ and $D$. **(5)**

A curve has equation

$$y = \mathrm{g}(x) \qquad x > 0$$

Using the answer to part (a),

(b) find $\mathrm{g}'(x)$. **(2)**

(c) Hence, explain why $\mathrm{g}'(x) > 3$ for all values of $x$ in the domain of g. **(1)**

(a) We are asked to do Partial Fractions

$$\frac{3x^3+8x^2-3x-6}{x(x+3)} = \underbrace{Ax+B+\frac{C}{x}+\frac{D}{x+3}}$$

manipulate this given form to get common denominators

$$\frac{3x^3+8x^2-3x-6}{x(x+3)} = \frac{(Ax+B)x(x+3)+C(x+3)+Dx}{x(x+3)}$$

Equate the numerators and substitute in values in order to get A, B and C.

$3x^3+8x^2-3x-6 \equiv (Ax+B)x(x+3)+C(x+3)+Dx$ M1

choose values to substitute in so that brackets become 0, to make your life easier!

$x=0 \Rightarrow 3(0)^3+8(0)^2-3(0)-6 = \cancel{(A(0)+B)(0)(0+3)}^{0} + C(0+3) + \cancel{D(0)}^{0}$

$-6 = 3C \Rightarrow$ $C=-2$

$x=-3 \Rightarrow 3(-3)^3+8(-3)^2-3(-3)-6 = \cancel{(A(-3)+B)(-3)(-3+3)}^{0} - \cancel{2(-3+3)}^{0} + D(-3)$

$-6=-3D \Rightarrow$ $D=2$ dM1A1

$x=2 \Rightarrow 3(2)^3+8(2)^2-3(2)-6 = (A(2)+B)(2)(5)-2(5)+2(2)$

$44 = 20A+10B-10+4$

$50 = 20A+10B$ Eq1

$x=1 \Rightarrow 3(1)^3+8(1)^2-3(1)-6 = (A+B)(1)(4)-2(4)+2$

$2 = 4A+4B-6$

$8 = 4A+4B$ Eq2 ddM1

Solve Simultaneously Eq1 and Eq2:

$50 = 20A+10B$

$8 = 4A+4B \;|\times -5|\; -40 = -20A-20B$ +

$10 = -10B \Rightarrow$ $B=-1.$ $A=3.$

Hence values found:

$A=3, B=-1, C=-2, D=2$ A1



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**Question 3 continued**

(b) In (a) we found that:

$$g(x) = 3x - 1 - \frac{2}{x} + \frac{2}{x+3}$$

$$g(x) = 3x - 1 - 2x^{-1} + 2(x+3)^{-1}$$

Differentiate

$$g'(x) = 3 + \frac{2}{x^2} - \frac{2}{(x+3)^2}$$

M1A1

(c) The given domain is $x > 0$.

When $x > 0$, $\frac{2}{x^2} > \frac{2}{(x+3)^2}$, so $\frac{2}{x^2} - \frac{2}{(x+3)^2} > 0$.

$$g'(x) = 3 + \underbrace{\frac{2}{x^2} - \frac{2}{(x+3)^2}}_{>0 \text{ when } x>0}$$

B1

Hence, $g'(x) > 3$.

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**Question 3 continued**

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**Question 3 continued**

Q3

**(Total 8 marks)**

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**4.** $\mathrm{f}(x) = \sqrt{1 - 4x^2}$ $\quad |x| < \frac{1}{2}$

(a) Find, in ascending powers of $x$, the first four non-zero terms of the binomial expansion of $\mathrm{f}(x)$. Give each coefficient in simplest form.

**(4)**

(b) By substituting $x = \frac{1}{4}$ into the binomial expansion of $\mathrm{f}(x)$, obtain an approximation for $\sqrt{3}$

Give your answer to 4 decimal places.

**(2)**

(a) Get the formula for the Binomial Expansion from the formula booklet:

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$ , Range of validity $|x|<1$

Substitute into the Formula:

$(1-4x^2)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(-4x^2) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(-4x^2)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-4x^2)^3 + \dots$

$= 1 - 2x^2 + \left(-\frac{1}{8}\right)(16x^4) + \left(\frac{1}{16}\right)(-64x^6) + \dots$ B1M1

$= 1 - 2x^2 - 2x^4 - 4x^6 + \dots$ A1A1

(b) Substitute $x = \frac{1}{4}$ into LHS:

$\left(1 - 4\left(\frac{1}{4}\right)^2\right)^{\frac{1}{2}} = \sqrt{\frac{3}{4}}$.

We are asked to find $\sqrt{3}$, hence we need to get rid of $\sqrt{\frac{1}{4}}$ from $\sqrt{\frac{3}{4}}$ by multiplying our expansion by 2:

$\sqrt{3} \approx 2\left(1 - 2\left(\frac{1}{4}\right)^2 - 2\left(\frac{1}{4}\right)^4 - 4\left(\frac{1}{4}\right)^6 + \dots\right)$ M1

$\sqrt{3} \approx \frac{887}{512} = 1.7324$ to 4dp A1

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**Question 4 continued**

Q4

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**5.**

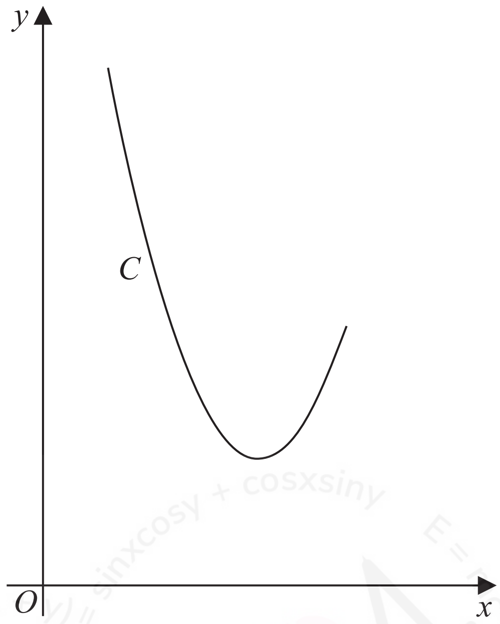


**Figure 1**

Figure 1 shows a sketch of the curve $C$ with parametric equations

$$x = 5 + 2\tan t \qquad y = 8\sec^2 t \qquad -\frac{\pi}{3} \leqslant t \leqslant \frac{\pi}{4}$$

(a) Use parametric differentiation to find the gradient of $C$ at $x = 3$

**(4)**

The curve $C$ has equation $y = \mathrm{f}(x)$, where f is a quadratic function.

(b) Find $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$, where $a$, $b$ and $c$ are constants to be found.

**(3)**

(c) Find the range of f.

**(2)**

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**Question 5 continued**

(a) ☆ Parametric differentiation:

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ get these by differentiating the parametrics separately.

$y = 8\sec^2 t$

$\frac{dy}{dt} = 16\sec t \cdot \sec t \tan t$

chain rule: multiply by derivative of bracket

$\frac{dy}{dx} = 16\sec^2 t \tan t$

$x = 5 + 2\tan t$

$\frac{dx}{dt} = 2\sec^2 t$

$\frac{dy}{dx} = \frac{16\sec^2 t\tan t}{2\sec^2 t}$

$\frac{dy}{dx} = 8\tan t.$ M1A1

Get the value of $t$ at $x=3$:

$3 = 5 + 2\tan t$

$-1 = \tan t$ dM1

Substitute $\tan t = -1$ into the found $\frac{dy}{dx}$:

$\frac{dy}{dx} = 8(-1)$

$\frac{dy}{dx} = -8$ gradient A1

(b) Manipulate the Identity $\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$ $\Rightarrow 1 + \tan^2 x = \sec^2 x$

From the given parametrics:

$y = 8\sec^2 t$

$\frac{y}{8} = \sec^2 t$

$x = 5 + 2\tan t$

$\left(\frac{x-5}{2}\right)^2 = \tan^2 t$

$1 + \tan^2 t = \sec^2 t$

$1 + \left(\frac{x-5}{2}\right)^2 = \frac{y}{8}$ M1A1

$8 + 8\frac{(x-5)^2}{4} = y$

A1 $y = 8 + 2(x-5)^2$ f(x) found

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**Question 5 continued**

(c) Use the given domain for t: $-\frac{\pi}{3} \leq t \leq \frac{\pi}{4}$.

Substitute into $y = 8\sec^2 t$ to get range:

$y = 8\sec^2(-\frac{\pi}{3}) = 32$ upper bound

$y = 8\sec^2(0) = 8$ lower bound

M1

Hence Range: $[8, 32]$ A1

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**Question 5 continued**

Q5

**(Total 9 marks)**





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**6.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

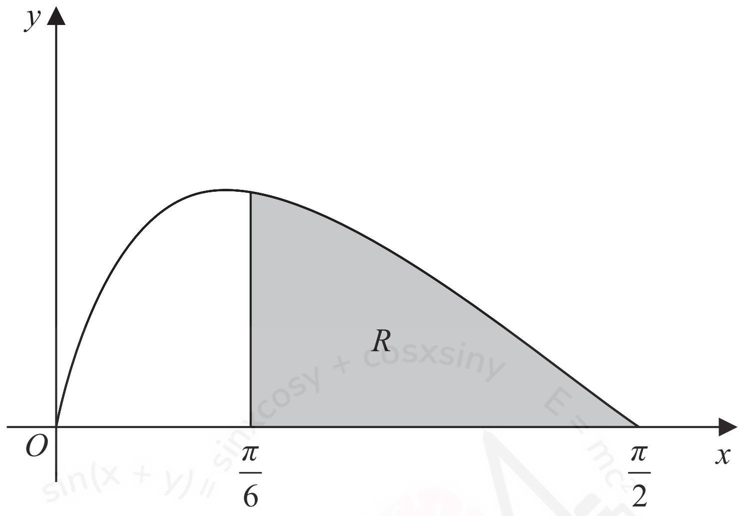


**Figure 2**

Figure 2 shows a sketch of the curve with equation

$$y = \frac{16\sin 2x}{(3 + 4\sin x)^2} \qquad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The region $R$, shown shaded in Figure 2, is bounded by the curve, the $x$-axis and the line with equation $x = \frac{\pi}{6}$

Using the substitution $u = 3 + 4\sin x$, show that the area of $R$ can be written in the form $a + \ln b$, where $a$ and $b$ are rational constants to be found.

**(7)**

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**Question 6 continued**

(a) To use the given substitution $u = 3 + 4\sin x$, we need to find new limits and $dx = \ldots$

New Limits:

| $x$ | | $u$ |
|---|---|---|
| $\frac{\pi}{6}$ | $\longrightarrow 3+4\sin\frac{\pi}{6} \longrightarrow$ | 5 |
| $\frac{\pi}{2}$ | $\longrightarrow 3+4\sin\frac{\pi}{2} \longrightarrow$ | 7 |

To get $dx$, differentiate $u = 3 + 4\sin x$

$$\frac{du}{dx} = 4\cos x$$

$$dx = \frac{du}{4\cos x}$$

B1

Substitute into the Integration:

$$y = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{16\sin 2x}{(3+4\sin x)^2}\, dx$$

$u$

$$= \int_5^7 \frac{16\sin 2x}{u^2} \times \frac{du}{4\cos x}$$

$$= \int_5^7 \frac{16(2\sin x\cos x)}{4u^2\cos x}\, du$$

apply double angle formula: $\sin 2A = 2\sin A\cos A$

$$= \int_5^7 \frac{\cancel{32}^{\,8}\sin x\cancel{\cos x}}{\cancel{4}u^2\cancel{\cos x}}\, du$$

$$= \int_5^7 \frac{8\sin x}{u^2}\, du \qquad \sin x = \frac{u-3}{4}$$

$$= \int_5^7 \frac{8\left(\frac{u-3}{4}\right)}{u^2}\, du$$

$$= \int_5^7 \frac{2u-6}{u^2}\, du$$

M1A1

$$= \int_5^7 \frac{2}{u} - \frac{6}{u^2}\, du$$

split the fraction and simplify

$$= \left[2\ln u + \frac{6}{u}\right]_5^7$$

dM1A1

$$= \left(2\ln 7 + \frac{6}{7}\right) - \left(2\ln 5 + \frac{6}{5}\right)$$

$$= 2\ln 7 - 2\ln 5 + \frac{6}{7} - \frac{6}{5}$$

$$= 2\ln\frac{7}{5} - \frac{12}{32}$$

apply ln law: $\ln a - \ln b = \ln\frac{a}{b}$

$$= -\frac{12}{32} + \ln\frac{49}{25}$$

apply ln law: $a\ln b = \ln b^a$

$\therefore$ area $R = -\frac{12}{32} + \ln\frac{49}{25}$

M1A1



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**Question 6 continued**

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**Question 6 continued**

Q6

**(Total 7 marks)**





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**7.** With respect to a fixed origin $O$,

- the line $l$ has equation $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix}$ where $\lambda$ is a scalar constant
- the point $A$ has position vector $9\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

Given that $X$ is the point on $l$ nearest to $A$,

(a) find

(i) the coordinates of $X$

(ii) the shortest distance from $A$ to $l$.
Give your answer in the form $\sqrt{d}$, where $d$ is an integer.

**(7)**

The point $B$ is the image of $A$ after reflection in $l$.

(b) Find the position vector of $B$.

**(2)**

(a) i. The smallest distance between X and A is perpendicular to $l$.

Hence $\vec{AX} \cdot \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix} = 0$. Use this fact to get the coordinates of X.

direction vector of $l$

$\vec{OX}$ is the general point on line $l$:

$$\vec{OX} = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix}$$

$\vec{OA}$ is given: $\vec{OA} = \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix}$

Get $\vec{AX}$: $\vec{AX} = \vec{OX} - \vec{OA}$

$$= \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix} \quad \text{M1}$$

Use $\vec{AX} \cdot \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix} = 0$ to get the value of $\lambda$ at X:

$$\begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix} = 0$$

Formula for dot product: $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = ad + be + cf$

$$-4(-5-4\lambda) - 3(5-3\lambda) + 5(-5+5\lambda) = 0$$

$$20 + 16\lambda - 15 + 9\lambda - 25 + 25\lambda = 0$$

$$50\lambda = -20 \Rightarrow \lambda = -\frac{4}{5}. \quad \text{dM1A1}$$

Hence $\vec{OX} = \begin{pmatrix} 4-4(-\frac{4}{5}) \\ 2-3(-\frac{4}{5}) \\ -3+5(-\frac{4}{5}) \end{pmatrix} = \begin{pmatrix} \frac{12}{5} \\ \frac{4}{5} \\ -1 \end{pmatrix} \Rightarrow X(\frac{12}{5}, \frac{4}{5}, -1)$ dM1A1

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**Question 7 continued**

ii. We need the magnitude of $\overrightarrow{AX}$.

$$\overrightarrow{AX} = \begin{pmatrix} -\frac{33}{5} \\ \frac{19}{5} \\ -3 \end{pmatrix}$$

Formula for Magnitude of a vector:

let $\overrightarrow{AB} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

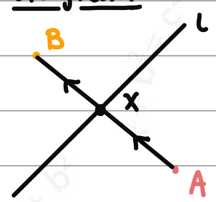
$$|\overrightarrow{AB}| = \sqrt{a^2 + b^2 + c^2}$$

$$|\overrightarrow{AX}| = \sqrt{\left(-\frac{33}{5}\right)^2 + \left(\frac{19}{5}\right)^2 + (-3)^2} \quad \text{M1}$$

$$= \sqrt{67} \quad \text{shortest distance}$$

A1

(b) Diagram




$\overrightarrow{AX} = \overrightarrow{XB}$. M1

Hence, $\overrightarrow{OB} = \overrightarrow{OX} + \overrightarrow{AX}$

$$= \begin{pmatrix} \frac{12}{5} \\ \frac{4}{5} \\ -1 \end{pmatrix} + \begin{pmatrix} -\frac{33}{5} \\ \frac{19}{5} \\ -3 \end{pmatrix}$$

$$\overrightarrow{OB} = \begin{pmatrix} -\frac{21}{5} \\ \frac{23}{5} \\ -4 \end{pmatrix} \quad \text{position vector}$$

A1

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**Question 7 continued**

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**Question 7 continued**

Q7

**(Total 9 marks)**

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**8.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

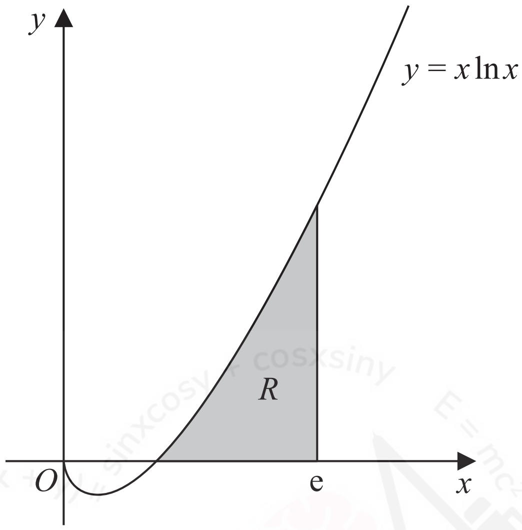


**Figure 3**

(a) Find $\int x^2 \ln x \, dx$

**(3)**

Figure 3 shows a sketch of part of the curve with equation

$$y = x\ln x \qquad x > 0$$

The region $R$, shown shaded in Figure 3, lies entirely above the $x$-axis and is bounded by the curve, the $x$-axis and the line with equation $x = \mathrm{e}$.

This region is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

(b) Find the exact volume of the solid formed, giving your answer in simplest form.

**(4)**

(a) $\int x^2 \ln x \, dx$

multiplication ∴ By Parts

$\int uv' \, dx = uv - \int u'v \, dx$

$u = \ln x \xrightarrow{\frac{d}{dx}} u' = \frac{1}{x}$

$v = \frac{1}{3}x^3 \xleftarrow{\int} v' = x^2$

$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \times \frac{1}{x} \, dx$ M1

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 \, dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$ dM1A1

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**Question 8 continued**

(b) Formula for Volume of Revolution around the $x$-axis:

$$V = \pi \int_a^b y^2 \, dx$$

where $a$ and $b$ are the points on the $x$-axis bounding the area

Get the lower bound:

$$0 = x \ln x$$

$$\ln x = 0$$

$$x = 1$$

Substitute in:

$$V = \pi \int_1^e (x \ln x)^2 \, dx$$

$$= \pi \int_1^e x^2 \ln^2 x \, dx$$

multiplication ∴ By Parts

$$\int uv' \, dx = uv - \int u'v \, dx$$

$$u = \ln^2 x \xrightarrow{\frac{d}{dx}} u' = 2\ln x \cdot \frac{1}{x}$$

$$v = \frac{1}{3}x^3 \xleftarrow{\int} v' = x^2$$

Chain Rule: multiply by derivative of the bracket

$$= \pi \left( \frac{1}{3}x^3 \ln^2 x - \int \frac{2\ln x}{x} \cdot \frac{1}{3}x^3 \, dx \right)_1^e \quad \text{M1}$$

$$= \pi \left( \frac{1}{3}x^3 \ln^2 x - \frac{2}{3} \underbrace{\int x^2 \ln x \, dx}_{\text{The result we got in (a)}} \right)_1^e$$

$$= \pi \left( \frac{1}{3}x^3 \ln^2 x - \frac{2}{3}\left( \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 \right) \right)_1^e \quad \text{A1}$$

$$= \pi \left( \left( \frac{1}{3}e^3 \cancel{\ln^2 e}^{\,1} - \frac{2}{3}\left( \frac{1}{3}e^3 \cancel{\ln e}^{\,1} - \frac{1}{9}e^3 \right) \right) - \left( \frac{1}{3}(1)^3 \cancel{\ln^2(1)}^{\,0} - \frac{2}{3}\left( \frac{1}{3}(1)^3 \cancel{\ln(1)}^{\,0} - \frac{1}{9}(1)^3 \right) \right) \right) \quad \text{dM1}$$

$$= \pi \left( \frac{1}{3}e^3 - \frac{2}{9}e^3 + \frac{2}{27}e^3 - \frac{2}{27} \right)$$

$$= \pi \left( \frac{5}{27}e^3 - \frac{2}{27} \right)$$

$$= \frac{5}{27}\pi e^3 - \frac{2}{27}\pi \quad \text{A1}$$



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**Question 8 continued**

Leave blank

**Question 8 continued**

Q8

**(Total 7 marks)**

Leave blank

**9.**

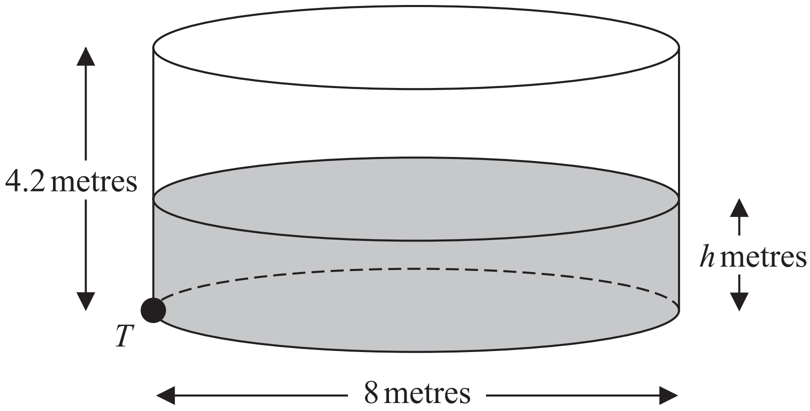


**Figure 4**

Figure 4 shows a cylindrical tank that contains some water.

The tank has an internal diameter of 8 m and an internal height of 4.2 m.

Water is flowing into the tank at a constant rate of $(0.6\pi)\,\text{m}^3$ per minute.

There is a tap at point $T$ at the bottom of the tank.

At time $t$ minutes after the tap has been opened,

- the depth of the water is $h$ metres
- the water is leaving the tank at a rate of $(0.15\pi h)\,\text{m}^3$ per minute

(a) Show that

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{12 - 3h}{320}$$

**(4)**

Given that the depth of the water in the tank is 0.5 m when the tap is opened,

(b) find the time taken for the depth of water in the tank to reach 3.5 m.

**(6)**

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**Question 9 continued**

(a) It's given that $\frac{dV}{dt} = \underbrace{0.6\pi}_{\text{flows in}} - \underbrace{0.15\pi h}_{\text{flows out}}$ B1

Formula for volume of cylinder:

$V = \pi r^2 h$

Volume of tank: $V = \pi(4)^2 h$

$V = 16\pi h$

$\therefore \frac{dV}{dh} = 16\pi$ B1

We want $\frac{dh}{dt}$: $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ given

$\frac{dh}{dt} = \frac{1}{16\cancel{\pi}}(0.6\cancel{\pi} - 0.15\cancel{\pi}h)$ M1

$= \frac{0.6 - 0.15h}{16}$

$\frac{dh}{dt} = \frac{12 - 3h}{320}$ hence shown

A1

Leave blank

**Question 9 continued**

(b) We need to solve the differential equation

$$\frac{dh}{dt} = \frac{12-3h}{320}$$

Separate Variables by grouping the like terms on one side

$$\int \frac{1}{12-3h}\,dh = \int \frac{1}{320}\,dt$$

Reverse Chain Rule: divide by derivative of the bracket

$$-\frac{1}{3}\ln|12-3h| = \frac{1}{320}t + c$$ M1A1

Use the given $t=0, h=0.5$ to get the value of $+c$:

$$-\frac{1}{3}\ln\left(12-\frac{3}{2}\right) = \frac{1}{320}(0) + c$$

$$c = -\frac{1}{3}\ln\frac{21}{2}$$

Hence the D.E. is: $-\frac{1}{3}\ln|12-3h| = \frac{1}{320}t - \frac{1}{3}\ln\frac{21}{2}$ M1A1

Substitute $h=3.5$ and solve for $t$:

$$-\frac{1}{3}\ln(12-3(3.5)) = \frac{1}{320}t - \frac{1}{3}\ln\frac{21}{2}$$

$$\frac{1}{3}\ln\frac{21}{2} - \frac{1}{3}\ln\left(\frac{3}{2}\right) = \frac{1}{320}t$$

$$320\left(\frac{1}{3}\ln\left(\frac{\frac{21}{2}}{\frac{3}{2}}\right)\right) = t$$ apply ln law $\ln a - \ln b = \ln\frac{a}{b}$

$$\frac{320}{3}\ln 7 = t$$ dM1

$t = 208$ min A1

Leave blank

**Question 9 continued**

**(Total 10 marks)**

Q9

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**10.** (a) A student's attempt to answer the question

"Prove by contradiction that if $n^3$ is even, then $n$ is even"

is shown below. Line 5 of the proof is missing.

Assume that there exists a number $n$ such that $n^3$ is even, but $n$ is odd.

If $n$ is odd then $n = 2p + 1$ where $p \in \mathbb{Z}$

So $n^3 = (2p + 1)^3$

$= 8p^3 + 12p^2 + 6p + 1$

$= 2(4p^3 + 6p^2 + 3p) + 1$, which is odd B1

This contradicts our initial assumption, so if $n^3$ is even, then $n$ is even.

Complete this proof by filling in line 5.

**(1)**

(b) Hence, prove by contradiction that $\sqrt[3]{2}$ is irrational.

**(5)**

Make an Assumption:
"Assume that $\sqrt[3]{2}$ is a rational number"

We want our assumption to be the opposite of what we are trying to prove.
In the Proof, we are trying to contradict the assumption

Since $\sqrt[3]{2}$ is rational, we can write it as:

$\sqrt[3]{2} = \frac{p}{q}$ where $p$ and $q$ are integers with no common factors. M1

$\therefore 2 = \frac{p^3}{q^3}$ cube both sides

$p^3 = 2q^3$, $p^3$ is even, hence $p$ is even and can be written as $p = 2m$, where $m$ is any integer. A1

$\therefore (2m)^3 = 2q^3$ M1

$8m^3 = 2q^3$

$\Rightarrow q^3 = 4m^3$, $q^3$ is even, $\therefore q$ is also even.

This means that p and q have a common factor 2, which is a contradiction to our initial statement. Hence proven by contradiction that $\sqrt[3]{2}$ is irrational.

A1A1

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**Question 10 continued**

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**Question 10 continued**

Q10

**(Total 6 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**